FLUID MECHANICS

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Potential Flow

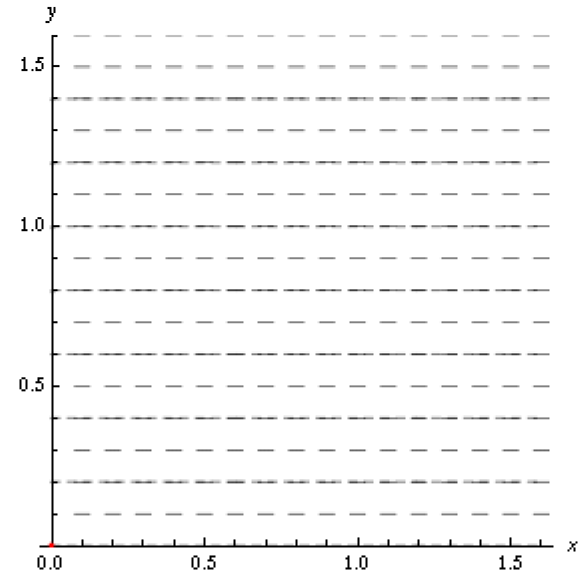
- **Behavior of a flowing fluid - solid boundaries.**
- **Influence of the wall is small**
  - **the shear stress may be negligible**
  - **one that is incompressible and has zero viscosity**
- ***potential flow is also called irrotational flow.***
- ***No dissipation of mechanical energy into heat.***

Concepts

Streamlines

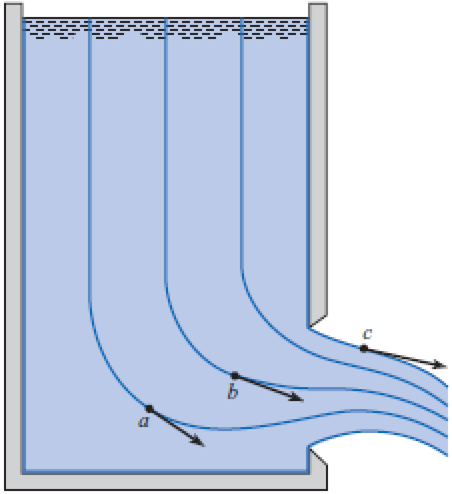
Pathlines

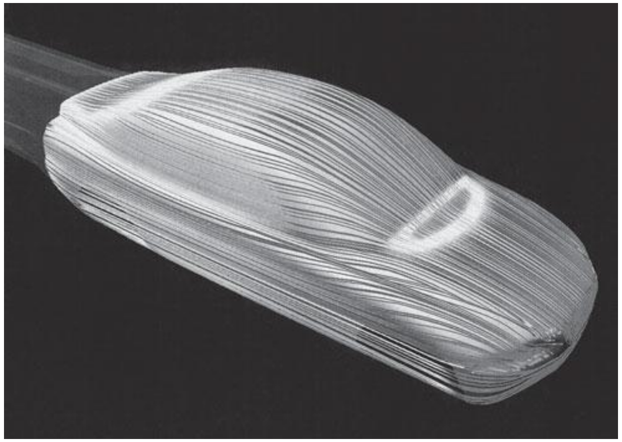
Streaklines



Streamline

- A streamline is a curve that is everywhere tangent to the instantaneous local velocity vector.
- The flow pattern provided by the streamlines is an instantaneous visualization of the flow field.





Pathline

A pathline is the actual path traveled by an individual fluid particle over some time period.

Streakline

A streakline is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.

Timeline

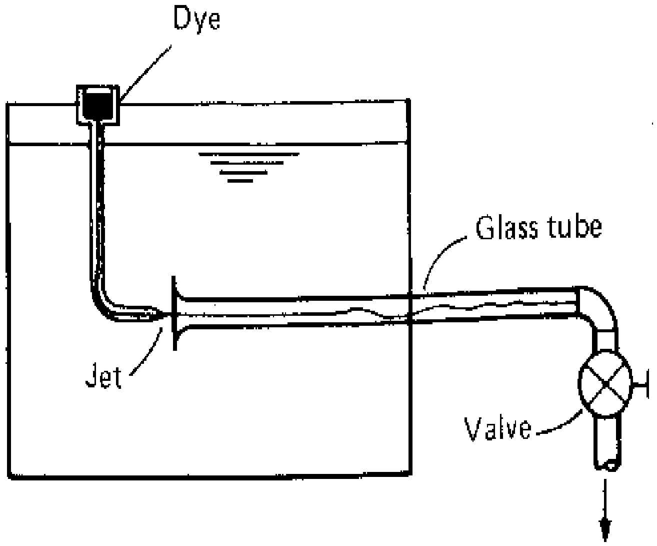
A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time.

Laminar vs Turbulent Flow

REYNOLDS EXPERIMENT

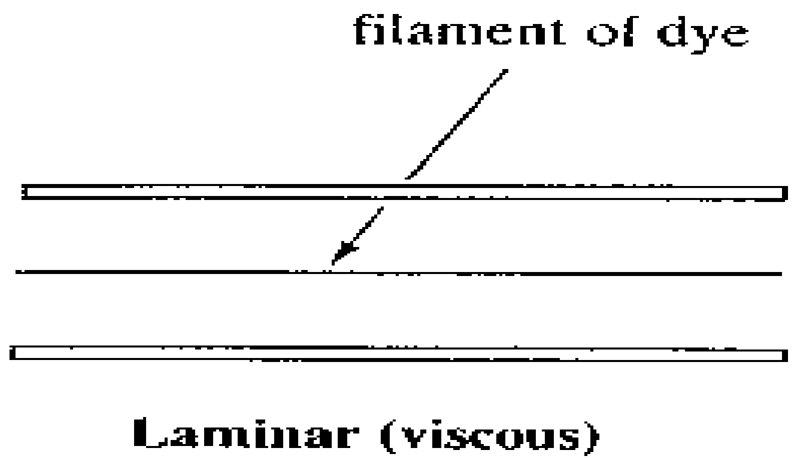
- **The distinction between the laminar and turbulent flow was first demonstrated in a classic experiment by Osborne Reynolds, reported in 1883.**

- **He used a tank with a pipe taking water from the center into which he injected a dye through a needle.**



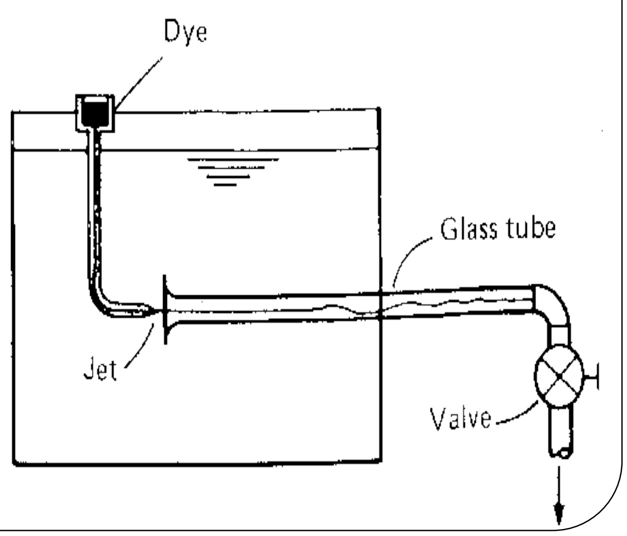
Reynolds Experiment Observations

- Reynolds found that, at low flow rates, the jet of colored water flowed intact along with the mainstream and no cross mixing occurred.
- The behavior of the color band showed clearly that the water was flowing in parallel straight lines and that the flow was laminar.

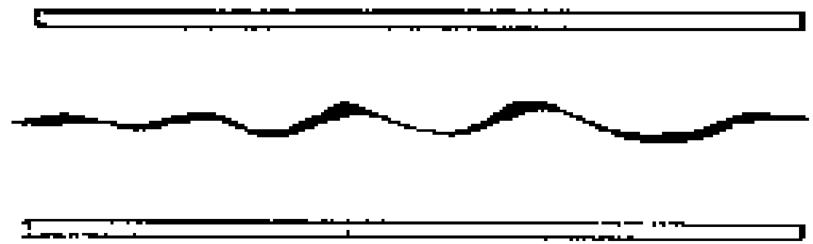


Reynolds Experiment Observations

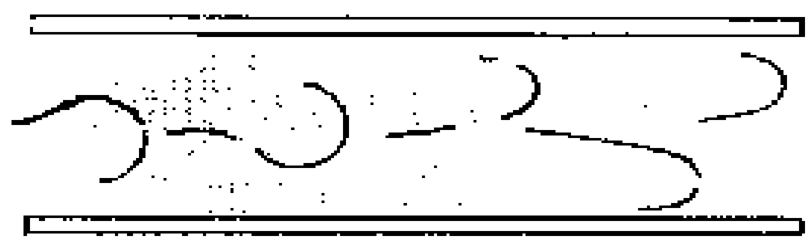
- **When the flow rate was increased, a velocity, called the critical velocity, was reached at which the thread of color became wavy and gradually disappeared, as the dye spread uniformly throughout the entire cross section of the stream of water.**
- **This behavior of the colored water showed that the water no longer flowed in laminar motion but moved erratically in the form of crosscurrents and eddies.**
- **This type of motion is turbulent flow.**



Reynolds Experiment Observations



Transitional



Turbulent

Reynolds Experiment: Summary

- **Laminar flow**
  - Re < 2000
  - 'low' velocity
  - Dye does not mix with water
  - Fluid particles move in straight lines
  - Simple mathematical analysis possible
  - Rare in practice in water systems.

- **Transitional flow**
  - 2000 < Re < 4000
  - 'medium' velocity
  - Dye stream wavers in water - mixes slightly.

Reynolds Experiment: Summary

- **<u>Turbulent flow</u>**
  - Re > 4000
  - 'high' velocity
  - Dye mixes rapidly and completely
  - Particle paths completely irregular
  - Average motion is in the direction of the flow
  - Cannot be seen by the naked eye
  - Changes/fluctuations are very difficult to detect. Must use laser.
  - Mathematical analysis very difficult - so experimental measures are used
  - Most common type of flow.

Concepts

$$N_{Re} = \frac{D\bar{V}\rho}{\mu} = \frac{D\bar{V}}{\nu}$$

In a pipe, when

$N_{Re} < 2100$	**(Laminar Flow)**
$2100 < N_{Re} < 4000$	**(Transition Flow)**
$N_{Re} > 4000$	**(Turbulent Flow)**

Reynolds No. for Non-Newtonian fluids

- For Power Law Fluids;

$$N_{\mathrm{Re},n} = 2^{3-n'} \left(\frac{n'}{3n' + 1} \right)^{n'} \frac{D^{n'} \rho \bar{V}^{2-n'}}{K'}$$

- Critical Reynolds number at which transition to turbulent flow begins;

$$N_{\mathrm{Re},nc} = 2100 \frac{(4n' + 2)(5n' + 3)}{3(3n' + 1)^2}$$

Concept of Boundary Layer

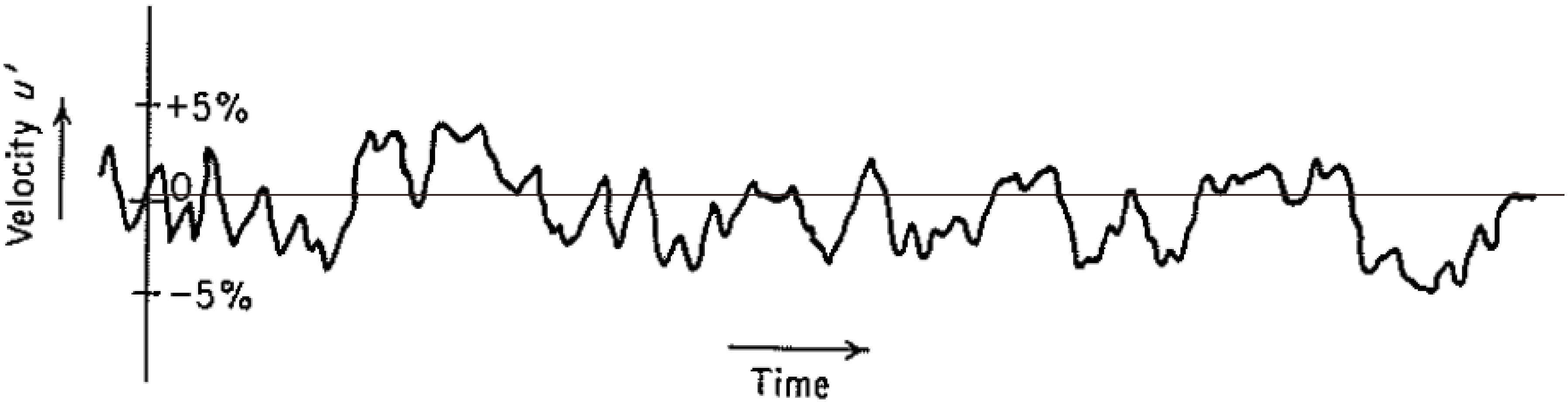
- **Boundary Layer**
  - **A boundary layer is defined as that part of a moving fluid in which the fluid motion is influenced by the presence of a solid boundary.**
- **Effect of Boundary**
  - **Velocity gradient and shear stress**
  - **Onset of turbulence**
  - **Formation and growth of boundary layer**
  - **Separation of boundary layer**

Nature of Turbulence

- **Wall Turbulence**
- **Free Turbulence**

- **Eddy formation**
- **Viscous dissipation**

Deviating velocities in turbulent flow



$$u_i = u + u' \qquad v_i = v' \qquad w_i = w'$$

u_i, v_i, w_i = instantaneous total velocity components in x, y, and z directions, respectively

u = constant net velocity of stream in x direction

u', v', w' = deviating velocities in x, y, and z directions, respectively

Deviating velocities in turbulent flow

$$p_i = p + p'$$

p_i = variable local pressure
p = constant average pressure as measured by ordinary manometers or pressure gauges
p' = fluctuating part of pressure due to eddies

Deviating velocities in turbulent flow

Because of the random nature of the fluctuations, the time averages of the fluctuating components of velocity and pressure vanish when averaged over a time period t_0 of the order of a few seconds. Therefore

$$\frac{1}{t_0}\int_0^{t_0} u'\,dt = 0 \qquad \frac{1}{t_0}\int_0^{t_0} w'\,dt = 0$$

$$\frac{1}{t_0}\int_0^{t_0} v'\,dt = 0 \qquad \frac{1}{t_0}\int_0^{t_0} p'\,dt = 0$$

The reason these averages vanish is that for every positive value of a fluctuation there is an equal negative value, and the algebraic sum is zero.

Deviating velocities in turbulent flow

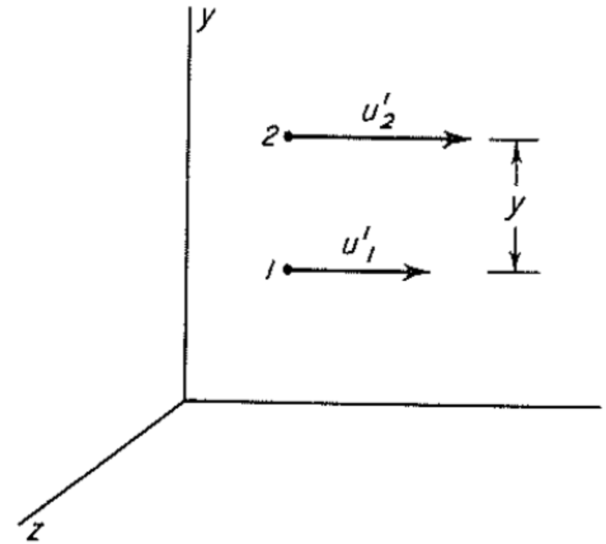
Although the time averages of the fluctuating components themselves are zero, this is not necessarily true of other functions or combinations of these components. For example, the time average of the mean square of any one of these velocity components is not zero. This quantity for component u' is defined by

$$\frac{1}{t_0}\int_0^{t^0} (u')^2\, dt = \overline{(u')^2}$$

Thus the mean square is not zero, since u' takes on a rapid series of positive and negative values, which, when squared, always give a positive product. Therefore $\overline{(u')^2}$ is inherently positive and vanishes only when turbulence does not exist.

Statistical Nature of Turbulence

- **Three deviating velocity components at single point as a function of time**
- **Deviating velocity at different positions over same time period**



Statistical Nature of Turbulence

- **Correlation of deviating velocity between eddies at two different points**

$$R_{u'} = \frac{\overline{u'_1 u'_2}}{\sqrt{\overline{(u'_1)^2}\,\overline{(u'_2)^2}}}$$

where u'_1 and u'_2 are the values of u' at stations 1 and 2, respectively.

- **Correlation of deviating velocity of eddies at same point**

$$R_{u'v'} = \frac{\overline{u'v'}}{\sqrt{\overline{(u')^2}\,\overline{(v')^2}}}$$

where u' and v' are measured at the same point at the same time

Intensity of Turbulence

- **Intensity of turbulent field refers to the speed of rotation of the eddies and the energy contained in an eddy of a specific size.**

- **Intensity is measured by the root-mean square of a velocity component.**

- **Intensity is usually expressed as a percentage of the mean velocity or as**

$$100\sqrt{\overline{(u')^2}}/u.$$

Scale of Turbulence

- **Scale of turbulence measures the size of the eddies.**

The scale of turbulence is based on correlation coefficients such as $R_{u'}$, measured as a function of the distance between stations. By determining the values of $R_{u'}$, as a function of y, the scale L_y of the eddy in the y direction is calculated by the integral

$$L_y = \int_0^\infty R_{u'}\, dy$$

Each direction usually gives a different value of L_y, depending upon the choice of velocity components used in the definition. For air flowing in pipes at 12 m/s, the scale is about 10 mm, and this is a measure of the average size of the eddies in the pipe.

Isotropic Turbulence

- **When the root-mean-square components of velocities are equal for all directions at a given point, the turbulence is said to be ISOTROPIC.**

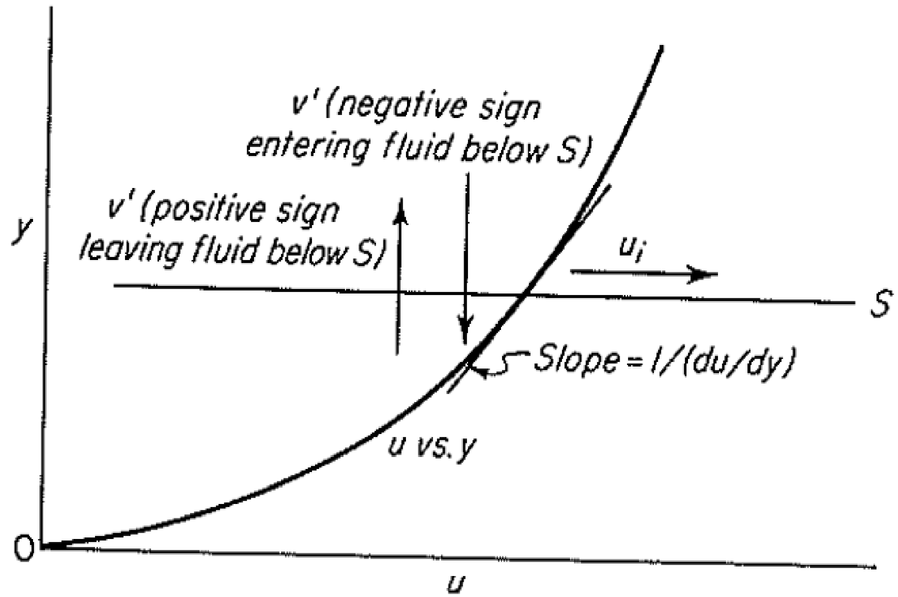
$$\overline{(u')^2} = \overline{(v')^2} = \overline{(w')^2}$$

- **Nearly isotropic turbulence exists when there is no velocity gradient, as at the centerline of a pipe or beyond the outer edge of a boundary layer.**

Reynolds Stress

- **Shear forces much larger than those occurring in laminar flow exist in turbulent flow**
- **The mechanism of turbulent shear depends upon the deviating velocities in anisotropic turbulence.**
- **Turbulent shear stresses are called Reynolds Stresses.**

Reynolds Stress



- **The momentum flux, after time averaging for all eddies, is a turbulent shear stress or *Reynolds Stress*.**

$$\tau_t g_c = \overline{\rho u' v'}$$

Eddy Viscosity

- **The relationship between shear stress and velocity gradient in a turbulent stream is used to define an eddy viscosity E_v .**

$$\tau_t g_c = E_v \frac{du}{dy}$$

Quantity E_v is analogous to μ, the absolute viscosity. Also, in analogy with the kinematic viscosity ν the quantity ε_M, called the *eddy diffusivity of momentum*, is defined as $\varepsilon_M = E_v/\rho$.

Eddy Viscosity

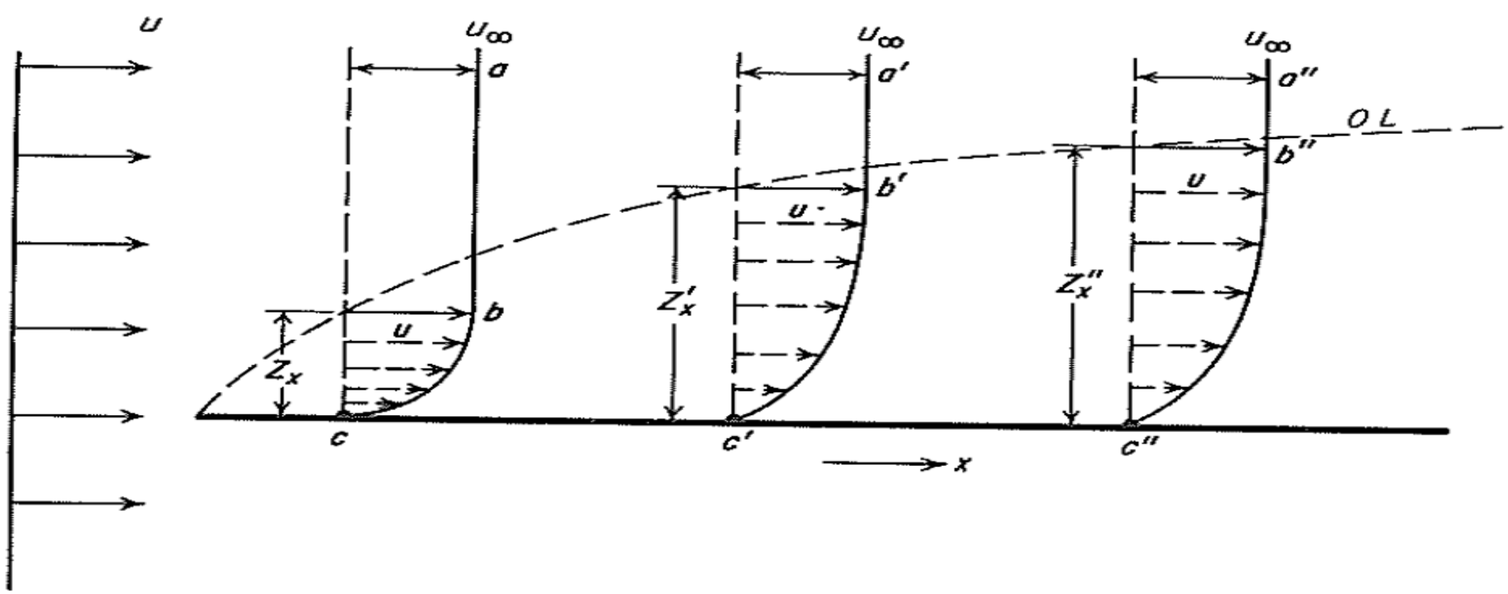
- **The total shear stress in a turbulent fluid is the sum of the viscous stress and the turbulent stress**

$$\tau g_c = (\mu + E_v)\frac{du}{dy}$$

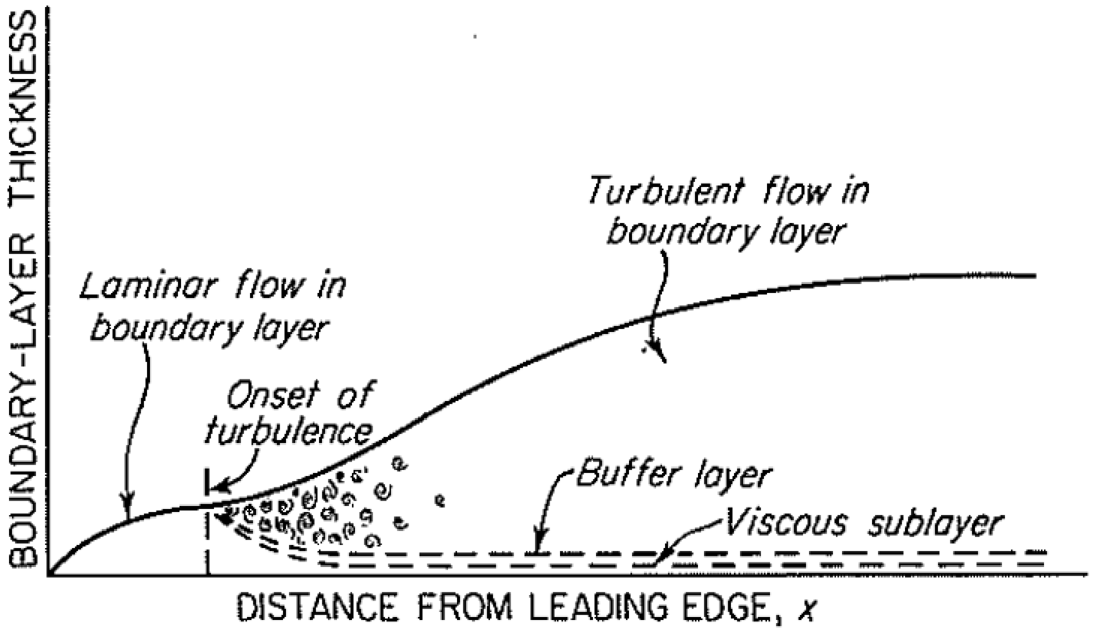
$$\tau g_c = (\nu + \varepsilon_M)\frac{d(\rho u)}{dy}$$

Flow in Boundary Layers

- **Boundary Layer**
  - **A boundary layer is defined as that part of a moving fluid in which the fluid motion is influenced by the presence of a solid boundary.**

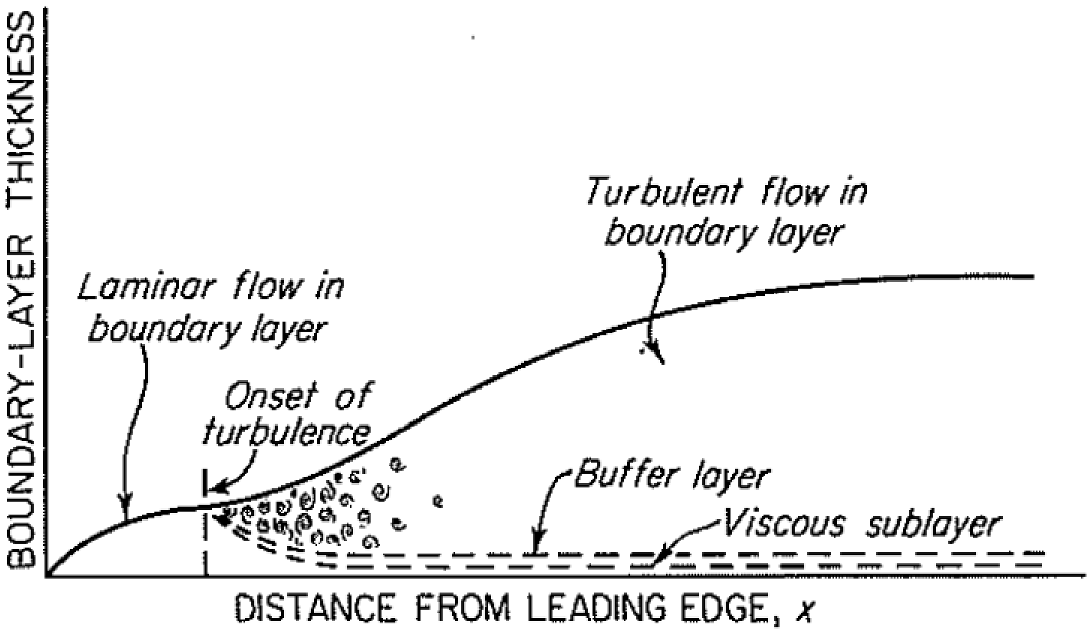


Laminar and Turbulent Flow in Boundary Layers



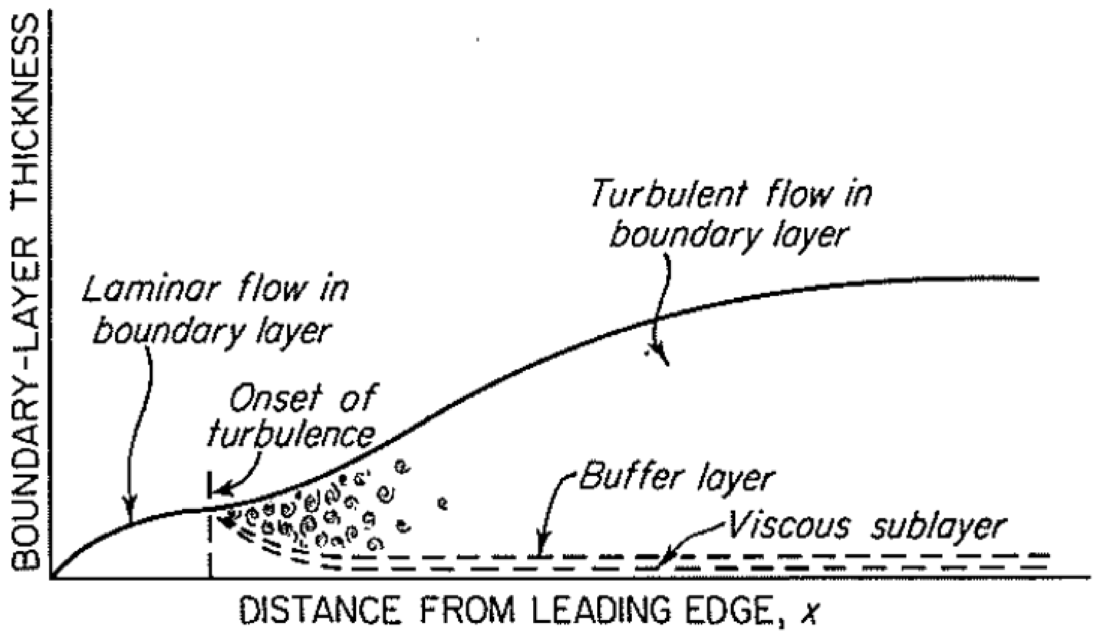
Near the leading edge of a flat plate immersed in a fluid of uniform velocity, the boundary layer is thin, and the flow in the boundary layer is entirely laminar. As the layer thickens, however, at distances farther from the leading edge, a point is reached where turbulence appears. The onset of turbulence is characterized by a sudden rapid increase in the thickness of the boundary layer, as shown in Fig.

Laminar and Turbulent Flow in Boundary Layers



- **Turbulent boundary layer consists of 3 zones:**
  - **Viscous sublayer**
  - **Buffer layer**
  - **Turbulent zone**

Laminar and Turbulent Flow in Boundary Layers



When flow in the boundary layer is laminar, the thickness Z_x of the layer increases with $x^{0.5}$, where x is the distance from the leading edge of the plate.[5] For a short time after turbulence appears, Z_x increases with $x^{1.5}$ and then, after turbulence is fully developed, with $x^{0.8}$.

Transition from Laminar and Turbulent Flow: Reynolds No

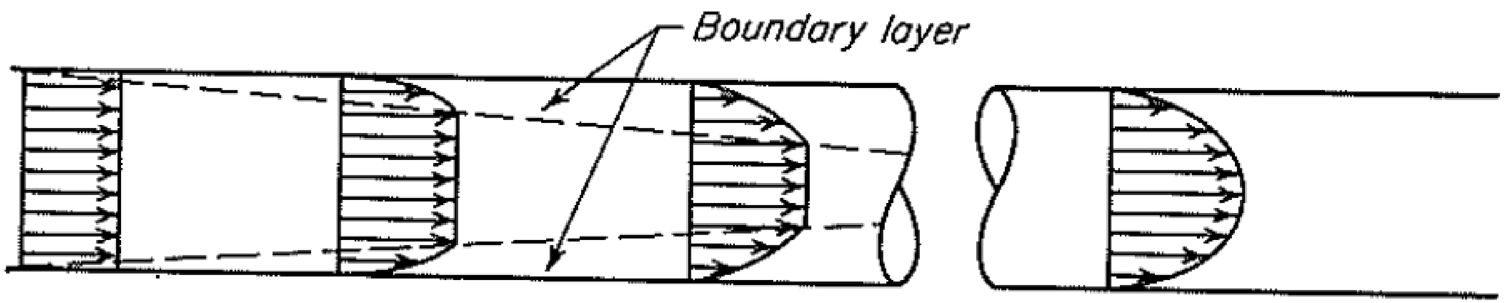
Transition from laminar to turbulent flow; Reynolds number.

$$N_{\mathrm{Re},x} = \frac{x u_\infty \rho}{\mu}$$

where x = distance from leading edge of plate
u_∞ = bulk fluid velocity
ρ = density of fluid
μ = viscosity of fluid

- **With parallel flow along a plate, turbulent flow first appears at a critical Reynolds number between about 10^5 and 3 E6**

Boundary Layer formation in straight tube



- **When the velocity distribution in the tube reaches its final form and remains unchanged during the remaining length of the tube, such flow with an unchanging velocity distribution is called fully developed flow.**

Boundary Layer formation in straight tube

The approximate length of straight pipe necessary for completion of the final velocity distribution is, for laminar flow,[4]

$$\frac{x_t}{D} = 0.05 N_{Re}$$

where x_t = transition length
D = diameter of pipe

Transition Length for Laminar and Turbulent flow:

The length of the entrance region of the tube necessary for the boundary layer to reach the center of the tube and for fully developed flow to be established is called the transition length.

Boundary Layer Separation and Wake formation